Further Analysis of Di-gluon Fusion Mechanism for the decays of $B \to K\eta'$

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Abstract

Di-gluon fusion mechanism might account for the large branching ratios of $B \to K\eta'$. But because we know little about the effective $\eta' gg$ vertex, there are large uncertainties in perturbative QCD estimations. In this paper, we try several kinds of $\eta' gg$ form factors and compare the numerical results with the experiment. We find that, though we know little about $\eta' gg$ form factor, if di-gluon fusion mechanism is important in $B \to K\eta'$, the branching ratios of the decays $B \to K^*\eta'$ would be around 10^{-5} which can be tested by future experiments.

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I. INTRODUCTION

Recently, CLEO collaboration have improved their previous measurements of $B \to K\eta'$ [1]:

$$\begin{aligned} \mathcal{B}(B^+ \to K^+\eta') &= (80^{+10}_{-9} \pm 8) \times 10^{-6} \\ \mathcal{B}(B^0 \to K^0\eta') &= (88^{+18}_{-16} \pm 9) \times 10^{-6}. \end{aligned} \tag{1}$$

These branching ratios are much larger than the estimations under the standard theorectical frame which is based on the effective Hamiltonian and general factorization approximation. Now it is commonly believed that these large branching ratios are due to the special properties of η', and several new mechanisms have been proposed to enhance the decay rates of $B \to \eta' K$ and $B \to \eta' X_s$. In the following we only discuss the standard model(SM) mechanisms because we think that the contribution of the SM should be carefully examined first.

Halperin and Zhitnisky [2] proposed an interesting mechanism: η' can be directly produced through $b \to \bar{c}cs \to \eta' s$ if $\langle \eta' | \bar{c}\gamma_\mu\gamma_5 c|0\rangle = -if^c_{\eta'}P^\mu_{\eta'} \neq 0$. But if this mechanism dominates, one easily find that

$$\begin{aligned} \frac{\mathcal{B}(B^0 \to \eta' K^{*0})}{\mathcal{B}(B^0 \to \eta' K^0)} &= \frac{|\langle \eta' | (\bar{c}c)_{V-A}|0\rangle\langle K^{*0}|(\bar{s}b)_{V-A}|B^0\rangle|^2}{|\langle \eta' | (\bar{c}c)_{V-A}|0\rangle\langle K^{0}|(\bar{s}b)_{V-A}|B^0\rangle|^2} \\ &= \frac{|2f^c_{\eta'}M_{K^*}A_0^{BK^*}(M^2_{\eta'})(\epsilon_{K^*}\cdot P_B)|^2}{|if^c_{\eta'}(M_B^2 - M_K^2)F_0^{BK}(M^2_{\eta'})|^2} \simeq \left(\frac{A_0^{BK^*}(M^2_{\eta'})}{F_0^{BK}(M^2_{\eta'})}\right)^2 \simeq 0.9, \end{aligned} \tag{2}$$

which is in contradiction with the stringent upper limit of $B^0 \to \eta' K^{*0}$ given by CLEO [1]:

$$\mathcal{B}(B^0 \to \eta' K^{*0}) < 2.0 \times 10^{-5}. \tag{3}$$

So this mechanism is unlikely to be dominant. Because the effective vertex of $b \to sgg$ in SM is very small, $b \to sgg \to s\eta'$ [3] is also impossible to account for the large $B \to \eta' K$ branching ratios. The authors of Ref [4] proposed that $b \to sg^*$ and $g^* \to g\eta'$ via QCD anomaly can account for the large semi-inclusive branching ratios of $B \to \eta' X_s$. CLEO [6] have measured the η' momentum spectrum in $B \to \eta' X_s$ and this measurement favors the mechanism of Ref [4]. But because it has an extra gluon in the final state, unless the gluon is soft and absorbed into the η' wave functions [12], it seems difficult to realize its contribution to two-body exclusive decay $B \to \eta' K$.

Motivated by the idea of Ref [4,5], the authors of Ref [7] proposed di-gluon fusion mechanism, which is depicted in Fig.1. Because both of the gluons are hard, it seems reasonable to give a perturbative QCD estimation. But because of our ignorance about the form factor of $\eta' gg$ vertex, there are large uncertainties in calculations. So in the following we would reanalyze di-gluon fusion mechanism in detail using several kinds of $\langle gg|\eta'\rangle$ form factors and compare our numerical results with experimental data.

II. ANALYSIS OF DI-GLUON FUSION MECHANISM

We first give a brief introduction on how to estimate contributions of di-gluon fusion mechanism in perturbative QCD.

$g-g-\eta'$ vertex can be parameterized as [9]:

$$\langle g^a g^b|\eta'\rangle = 4\delta^{ab} g_s^2 A_{\eta'} F(k_1^2, k_2^2)\epsilon_{\mu\nu\rho\sigma} k_1^\mu k_2^\nu \epsilon_1^\rho \epsilon_2^\sigma, \tag{4}$$

where $A_{\eta'}$ is a constant and can be extracted from $J/\Psi \to \gamma\eta'$ or $\eta' \to \gamma\gamma$, g_s is QCD coupling constant, $F(k_1^2, k_2^2)$ is $\langle gg|\eta'\rangle$ form factor and $F(0,0)=1$. Neglecting the transverse momentum of quarks and taking the wave functions for B and K meson as

$$\Psi_B = \frac{1}{\sqrt{2}}\frac{I_C}{\sqrt{3}}\phi_B(x)(\not{p}+m_B)\gamma_5, \quad \Psi_K = \frac{1}{\sqrt{2}}\frac{I_C}{\sqrt{3}}\phi_K(y)\not{q}\gamma_5, \tag{5}$$

where I_C is an identity in color space, and the integration of the distribution amplitude is related to the meson decay constant,

$$\int \phi_B(x)dx = \frac{f_B}{2\sqrt{6}}, \qquad \int \phi_K(y)dy = \frac{f_K}{2\sqrt{6}}. \tag{6}$$

In this paper, we take the distribution amplitudes as [10]:

$$\phi_B(x) = \frac{1}{2\sqrt{6}}\delta(x-\epsilon_B), \qquad \phi_K(y) = \sqrt{\frac{3}{2}}y(1-y). \tag{7}$$

Then we can write the amplitude of Fig.1 as

$$\mathcal{M} = \int dxdy\phi_B(x)\frac{Tr[\gamma_5\not{q}\Gamma_\mu(\not{p}+m_B)\gamma_5\gamma_\nu]4\epsilon^{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}A_{\eta'}F(k_1^2,k_2^2)C_F g_s^3}{\sqrt{2}\sqrt{2}k_1^2k_2^2}\phi_K(y), \tag{8}$$

where Γ_μ is the effective $b\to sg$ vertex [8]

$$\Gamma_\mu^a = \frac{G_F}{\sqrt{2}}\frac{g_s}{4\pi^2}V_{is}^*V_{ib}\bar{s}t^a\left\{F_1^i(k_1^2\gamma_\mu - k_{1\mu}\not{k}_1)\frac{1-\gamma_5}{2} - F_2^i i\sigma_{\mu\nu}k_1^\nu m_b\frac{1+\gamma_5}{2}\right\}b, \tag{9}$$

and the color factor $C_F = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}Tr[t^at^b]\delta^{ab} = \frac{4}{3}$. Finally we obtain

$$\begin{aligned}\mathcal{M} = &\frac{G_F}{\sqrt{2}}\alpha_s^2 A_{\eta'}C_F 32V_{is}^*V_{ib}\int dxdy\phi_B(x)\phi_K(y)F(k_1^2,k_2^2)\\ &\times\frac{F_1^ik_1^2[(p\cdot k_1)(q\cdot k_2)-(p\cdot k_2)(q\cdot k_1)]+F_2^im_Bm_b[(q\cdot k_2)k_1^2-(q\cdot k_1)(k_1\cdot k_2)]}{k_1^2k_2^2}.\end{aligned} \tag{10}$$

In the above integral, there are two singularities from the gluon propagators and sometimes there would be another singularity from the $\langle gg|\eta'\rangle$ form factor (For example, $F(k_1^2,k_2^2) = \frac{m_{\eta'}^2}{2(k_1\cdot k_2)}$). We add a small pure imaginary number $i\epsilon$ ($\epsilon > 0$) to the denominator to get a convergent integral.

To evaluate the numerical result of Eq.(10), we take the form factors F_1 and F_2 according to ref [4,17] as

$$F_1 = \frac{4\pi}{\alpha_s}(C_4+C_6), \qquad F_2 = -2C_8. \tag{11}$$

Where C_i are wilson coefficients at the NLL level.(Because QCD corrections maybe large, we do not take [5,8] $F_1 \simeq -5$ and $F_2 \simeq 0.2$ which are derived from the SM without QCD corrections.) It is always subtle to choose the scale of α_s, because the average momenta squared of the gluons are

$$\langle k_1^2 \rangle \simeq 12\ GeV^2 \qquad \langle k_2^2 \rangle \simeq 1\ GeV^2 \tag{12}$$

we prefer to choose $\alpha_s = \sqrt{\alpha_s(k_1^2)\alpha_s(k_2^2)} = 0.28$ though we also give the branching ratios when $\alpha_s = \alpha_s(k_1^2) = 0.21$ or $\alpha_s = \alpha_s(k_2^2) = 0.38$ in Tab.1 and Tab.2.

As to the $\langle gg|\eta' \rangle$ form factor $F(k_1^2, k_2^2)$, because of our complete ignorance there are large uncertainties in the numerical evaluation of Eq.(10). In the following we try several kinds of form factors.

In Ref [4,5], the authors have assumed that q^2-dependence of the form factor is weak and as an approximation they take form factor as a constant to explain large semi-inclusive decay $B \to X_s \eta'$. The difference between Ref [4] and Ref [5] is that the running of $\alpha_s(\mu)$ with the scale is considered in [5] but not in [4]. We also use their ansatz to estimate exclusive decays $B \to K\eta'$ and $B \to K^*\eta'$. Using Eq.(10), we give numerical results in Tab.1. From the Table it seems that the constant form factor can account for the large branching ratios of $B \to K\eta'$, but unfortunately we would get much larger branching ratios of $B \to K^*\eta'$, which is strongly in contradiction with the CLEO's measurements [1] (see, for instance, the case of $\epsilon_B = 0.06$ in Tab.1).

From Eq.(10), we notice that the amplitude must be integrated over a wide range of k_1^2 and k_2^2, therefore the effects of k_1^2, k_2^2 dependence of $F(k_1^2, k_2^2)$ must be taken into account.

The authors of Ref [7] choose the form factor as $F(k_1^2, k_2^2) = \frac{m_{\eta'}^2}{2(k_1 \cdot k_2)}$ because such a form factor works well in $J/\Psi \to \gamma\eta'$ [14]. We have followed their calculations and it seems that their numerical results are overestimated than ours(in Tab.2) by a factor about three. From Tab.2, we can see that our estimations of $\mathcal{B}(B \to \eta' K)$ are about 10^{-5}, but when considering that the standard theoretical frame (based on effective Hamiltonian and general factorization approximation) can give [12] $\mathcal{B}(B \to \eta' K) \simeq 3.6 \times 10^{-5}$, it is still possible to account for the experimental data only if the contributions from di-gluon fusion mechanism and the standard theoretical frame constructively interfere. If this is true, then because the contributions to $\mathcal{B}(B \to \eta' K^*)$ from the standard theoretical frame are negligible, the di-gluon fusion mechanism would be dominant in $B \to \eta' K^*$, *i.e.*, $\mathcal{B}(B \to \eta' K^*) \sim 10^{-5}$ which can be tested by future measurements of CLEO or B factories.

Kagan and Petrov [11] proposed a model of the $g - g - \eta'$ vertex in which a pseudoscalar current is coupled to two gluons through quark loops. Their perturbative calculations yield a form factor:

$$\begin{aligned} F(k_1^2, k_2^2) \propto & \sum_{f=u,d,s} a_f m_f \\ & \times \int_0^1 dx \int_0^{1-x} \frac{dy}{m_f^2 - (1-x-y)(xk_1^2 + yk_2^2) - xym_{\eta'}^2 - i\epsilon}, \end{aligned} \tag{13}$$

where we take $a_u = a_d = 1$, $a_s = 2$ and normalize $F(k_1^2, k_2^2)$ with the normal condition $F(0,0) = 1$. We use this form factor in Eq.(10) with $\epsilon_B = 0.06$ and get

$$\mathcal{B}(B \to \eta' K) = 1.28 \times 10^{-7}. \tag{14}$$

which is too small to account for the experiments.

In Ref [16], the authors calculate the transition form factor in π^0 coupling to $\gamma^*\gamma^*$ in the frame of a perturbative QCD based on the modified factorization formula. They find that numerically their results are extremely similar to that obtained by applying the interpolation procedure in the manner of Brodsky and Lepage in the case of two off-shell photons:

$$F_{\pi^0\gamma^*\gamma^*}(k_1^2, k_2^2) \propto (1 - \frac{X^2}{\Lambda_\pi^2})^{-1}, \tag{15}$$

with

$$X^2 = \frac{(k_1^2 - k_2^2)^3}{k_1^4 - k_2^4 - 2k_1^2 k_2^2 \ln(k_1^2/k_2^2)} \tag{16}$$

and $\Lambda_\pi = 2\pi f_\pi = 0.83GeV$. We assume that the structure of the transition of $\eta' \to g^* g^*$ is similar to that of $\pi^0 \to \gamma^*\gamma^*$:

$$F(k_1^2, k_2^2) = (1 - \frac{X^2}{\Lambda_{\eta'}^2})^{-1}, \tag{17}$$

where we approximate $\Lambda_{\eta'} = \Lambda_\pi$. By using this form factor, we calculate the branching ratios of $B \to K\eta'$ which are listed in Tab.3. The results are about fifty times smaller than the experimental data.

In experimental fit, pole approximation is often used to fit the momentum square dependence of form factor [15]. As a try, we also assume $F(k_1^2, k_2^2) = \frac{1}{(1-k_1^2/m_{\eta'}^2)(1-k_2^2/m_{\eta'}^2)}$ for our calculations, we list the results in Tab.4. And we can see that this kind of form factor would make the digluon fusion mechanism completely negligible in $B \to \eta' K$.

To examine the validity of perturbative QCD in the above caculations, or in other words, whether the amplitude calculated by perturbative QCD is dominated by hard gluon contributions, we set cut on k_2^2 and compare the results with those without cut. Taking $\Lambda_{QCD} \simeq 200MeV$, we list the numerical results in Tab.V. We can see that hard gluon contributions are really dominant in the case of constant form factor. But if we take form factor as $F(k_1^2, k_2^2) = \frac{m_{\eta'}^2}{2k_1 \cdot k_2}$, hard gluon contributions are small, this does not mean that non-perturbative contributions are dominant in this case. This is due to the fact that the singularity in the form factor is accidently close to the other singularities in k_1^2 and k_2^2 and then enhance the contributions of soft gluon.

III. REMARKS AND CONCLUSIONS

The authors of Ref [7] proposed digluon fusion mechanism to explain the large branching ratios of $B \to \eta' K$, but because of our ignorance about effective $\eta' gg$ vertex, there are large uncertainties in perturbative QCD estimations. We try several kinds of $\langle gg|\eta'\rangle$ form factors, and through our calculations, we find that constant form factor is consistent with the data of $B \to \eta' K$, but inconsistent with the data of $B \to \eta' K^*$ in some cases of different α_s or ϵ_B

. if we take $F(k_1^2, k_2^2) = \frac{m_{\eta'}^2}{2k_1 \cdot k_2}$ as the authors of Ref [7] have done, di-gluon fusion mechanism is important in $B \to \eta' K$ but not dominant. As a consequence, we could anticipate that $\mathcal{B}(B \to \eta' K^*)$ would be about 10^{-5} which can be tested by future experiments. We also try the other three kinds of form factors and they all give very small contributions to the branching ratios.

We conclude that, though we know little about $\langle gg|\eta'\rangle$ form factor, if di-gluon fusion mechanism is important in $B \to \eta' K$, the branching ratios of $B \to \eta' K^*$ would be definitely around 10^{-5}.

IV. ACKNOWLEDGEMENTS

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REFERENCES

[1] CLEO Collaboration, S.J. Richichi et al., CLEO-CONF-99-12, Aug 1999. (preprint hep-ex/9908019).

[2] I. Halperin and A. Zhitnisky, Phys. Rev. Lett.**80**, 438(1998).

[3] A. Datta, X.G. He and S. Pakvasa, Phys. Lett. B **419**, 369(1998).

[4] D. Atwood and A. Soni, Phys.Lett.B **405**, 150(1997).

[5] W. S. Hou and B. Tseng, Phys.Rev.Lett. **80**, 434(1998).

[6] CLEO Collaboration, T.E. Browder *et al.*, Phys. Rev. Lett.**81**, 1786(1998).

[7] D. Du, C. S. Kim and Y. Yang, Phys.Lett.B **426**, 133(1998).

[8] W. S. Hou, Nucl. Phys. B **308**, 561(1988).

[9] F. E. Close, G. R. Farrar and Z. Li, Phys.Rev.D **55**, 5749(1997).

[10] A. Szczepaniak, E. Henley and S. J. Brodsky, Phy.Lett.B **152**, 380(1990);
C. E. Carlson and J. Milana, Phys.Rev.D **49**, 5908(1994); Phys.Lett.B **301**, 237(1993);
H. Simma and D. Wyler, Phys.Lett.B **272**, 395(1991);
C. D. Lü and D. X. Zhang, Phys.Lett.B **400**, 188(1997).

[11] A. L. Kagan and A. A. Petrov, preprint hep-ph/9707354.

[12] H.Y. Cheng and B. Tseng, Phys. Lett. B **415**, 263(1997); H.Y. Cheng, hep-ph/9712244.

[13] G. P. Lepage and S. J. Brodsky, Phys. Rev. Lett. **43**, 545(1979); Phys. Rev. D **22**, 2157(1980); Phys. Rev. D **24**, 1808(1981).

[14] J. G. Köner, J. H. Kühn, M. Krammer and H. Schneider, Nucl. Phys. B **229**, 115(1983).

[15] CLEO Collaboration, J. Gronberg *et al.*, Phys.Rev.D **57**, 33(1998); L3 Collaboration, M. Acciarri *et al.*, Phys.Lett.B **418**, 399(1998).

[16] Saro Ong, Phys.Rev.D **52**, 3111(1995); P. Kessler and S. Ong, Phys. Rev. D **48**, 2974(1993).

[17] Xiao-Gang He and Guey-Lin Lin, preprint hep-ph/9812248.

TABLES

ϵ_B	0.05			0.06			0.07			Exp. [1]
α_s	0.21	0.28	0.38	0.21	0.28	0.38	0.21	0.28	0.38	
$\mathcal{BR}(B^- \to K^- \eta')$	1.99	6.60	21.3	1.32	4.40	14.2	0.94	3.13	10.1	$8.0^{+1.0}_{-0.9} \pm 0.8$
$\mathcal{BR}(\bar{B}^0 \to \bar{K}^0 \eta')$	2.06	6.88	22.2	1.38	4.59	14.8	0.98	3.26	10.5	$8.8^{+1.8}_{-1.6} \pm 0.9$
$\mathcal{BR}(B^- \to K^{*-} \eta')$	4.04	13.5	43.4	2.40	8.00	25.8	1.59	5.30	17.1	< 8.7
$\mathcal{BR}(\bar{B}^0 \to \bar{K}^{*0} \eta')$	4.19	14.0	45.1	2.49	8.31	26.8	1.66	5.52	17.8	< 2.0

TABLE I. The branching ratios of $B \to K\eta'$ and $B \to K^*\eta'$ in unit of 10^{-5} by using constant form factor $F(k_1^2, k_2^2) = 1$. Where $\alpha_s(k_1^2 = 12GeV^2) = 0.21$, $\alpha_s(k_2^2 = 1GeV^2) = 0.38$ and $\alpha_s = \sqrt{\alpha_s(k_1^2)\alpha_s(k_2^2)} = 0.28$.

ϵ_B	0.05			0.06			0.07			Exp. [1]
α_s	0.21	0.28	0.38	0.21	0.28	0.38	0.21	0.28	0.38	
$\mathcal{BR}(B^- \to K^- \eta')$	0.60	2.07	6.42	0.47	1.63	5.05	0.39	1.34	4.15	$8.0^{+1.0}_{-0.9} \pm 0.8$
$\mathcal{BR}(\bar{B}^0 \to \bar{K}^0 \eta')$	0.62	2.15	6.67	0.49	1.70	5.27	0.40	1.39	4.31	$8.8^{+1.8}_{-1.6} \pm 0.9$
$\mathcal{BR}(B^- \to K^{*-} \eta')$	0.91	3.17	9.83	0.67	2.31	7.16	0.54	1.89	5.86	< 8.7
$\mathcal{BR}(\bar{B}^0 \to \bar{K}^{*0} \eta')$	0.95	3.30	10.2	0.69	2.40	7.44	0.57	1.97	6.11	< 2.0

TABLE II. The branching ratios of $B \to K\eta'$ and $B \to K^*\eta'$ in unit of 10^{-5} by using the form factor $F(k_1^2, k_2^2) = \frac{m_{\eta'}^2}{2(k_1 \cdot k_2)}$. Where $\alpha_s(k_1^2 = 12GeV^2) = 0.21$, $\alpha_s(k_2^2 = 1GeV^2) = 0.38$ and $\alpha_s = \sqrt{\alpha_s(k_1^2)\alpha_s(k_2^2)} = 0.28$.

ϵ_B	0.04	0.05	0.06	0.07	0.08
$\mathcal{B}(B^- \to K^- \eta')$	2.31	1.71	1.34	1.09	0.92

TABLE III. The branching ratios of $B \to K\eta'$ in unit of 10^{-6} by using the form factor $F(k_1^2, k_2^2) = (1 - X^2/\Lambda^2)^{-1}$.

ϵ_B	0.04	0.05	0.06	0.07	0.08
$\mathcal{BR}(B^- \to K^- \eta')$	2.01	1.30	0.91	0.67	0.52
$\mathcal{BR}(\bar{B}^0 \to \bar{K}^0 \eta')$	2.09	1.35	0.95	0.70	0.53

TABLE IV. The branching ratios of $B \to K\eta'$ in unit of 10^{-6} by using the form factor $F(k_1^2, k_2^2) = \frac{1}{(1-k_1^2/m_{\eta^{‘}})(1-k_2^2/m_{\eta'})}$.

Decay Modes	$B^- \to \eta' K^-$		$\bar{B}^0 \to \eta' K^{*-}$	
Form Factor	$F(k_1^2, k_2^2) = \frac{m_{\eta'}^2}{2(k_1 \cdot k_2)}$	$F(k_1^2, k_2^2) = 1$	$F(k_1^2, k_2^2) = \frac{m_{\eta'}^2}{2(k_1 \cdot k_2)}$	$F(k_1^2, k_2^2) = 1$
Λ_{QCD}^2	71%	100%	72%	94%
$4\Lambda_{QCD}^2$	45%	93%	43%	84%
$9\Lambda_{QCD}^2$	22%	71%	17%	58%

TABLE V. When $\epsilon_B = 0.06$, the ratio of ‘hard contricbution’ to di-gluon fusion mechanism with k_2^2 cut of Λ_{QCD}^2, $4\Lambda_{QCD}^2$ and $9\Lambda_{QCD}^2$.

FIGURES

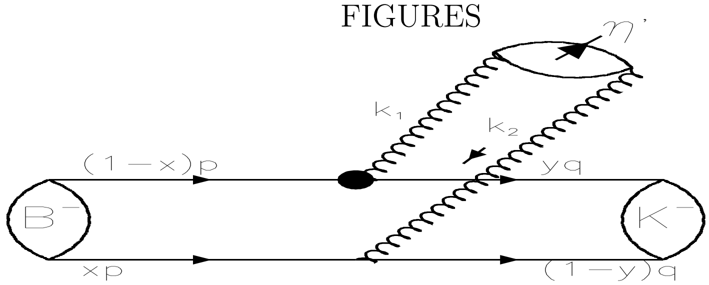

FIG. 1. *The diagram for di-gluon fusion mechanism.*